Hamming Codes in the Context of DNA Self Assembly

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Abstract. In the context of DNA self assembly, error detection and correction pose to be critical problems from the viewpoint of any practical implementation. So far, the best known algorithms for this purpose tend to require an increasing amount of tile types as the size of the system grows and lack modularity. Currently known algorithms include *tile proofreading* by Winfree and Bekbolatov [4] in which an original $n \ x \ m$ assembly is mapped to a larger, redundant system replacing individual tiles by a $k \ x \ k$ block; *snake tiles* by Goel and Chen [2] in which each tile in the original system corresponds to four tile blocks with all internal glues chosen to be unique; and, some other methods which leverage biological processes such as strand invasion or the use of restriction and ligation enzymes instead of the simple operations permitted by the Wang tile model. In our work, we construct and simulate a Wang tiling system of $O(1)$ capable of detecting errors in a given (L, D) Hamming code represented as a 'seed' column where the length of the code is L, and the number of data bits is D. In order to avoid keeping track of parity and data bit positions in the Hamming code separately, we exploit the correspondence between the parity bit positions in the Hamming code and the columns of the binary counter. We replace each of the 8 types of tiles in the binary counter tile system constructed by Goel et. al. [1] by 4 types of tiles capable of transmitting parity information toward the next tile, and discriminating between taking into account or skipping a data bit depending on the parity bit being checked resulting in a 32 tile system shown in Figure 1. We replace the double strength glues in the seed row of the binary counter by single strength glues to ensure that the system will stop growing after exactly L layers, since that is when we see the output. Thus, using just $O(1)$ tile types this system self assembles into a $log(L - D) \times L$ rectangular module at temperature 2 with only single strength glues. Any other module which outputs a L bit Hamming code can then be attached to the eastern edge of this module to check the correctness of the output. The correctness of each parity check from the Hamming code is shown as one of two glues on the northern edge of the system; see Figure 2 for an example. The robustness of the system is demonstrated via XGrow [3] simulations. In simulation we were unable to achieve erroneous output.

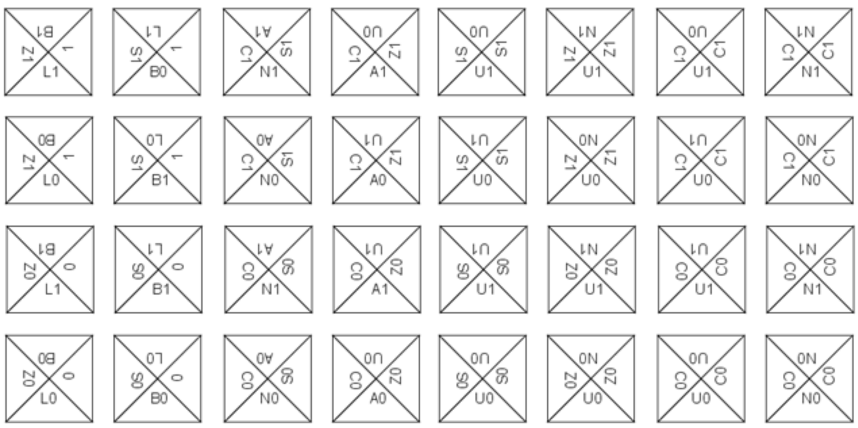

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Figure 1 - The TAK Tile System – A 32 tile system for error detection in Hamming code sequences at temperature 2, with glues B0 and B1 being strength 2 and the rest of the glues of strength 1

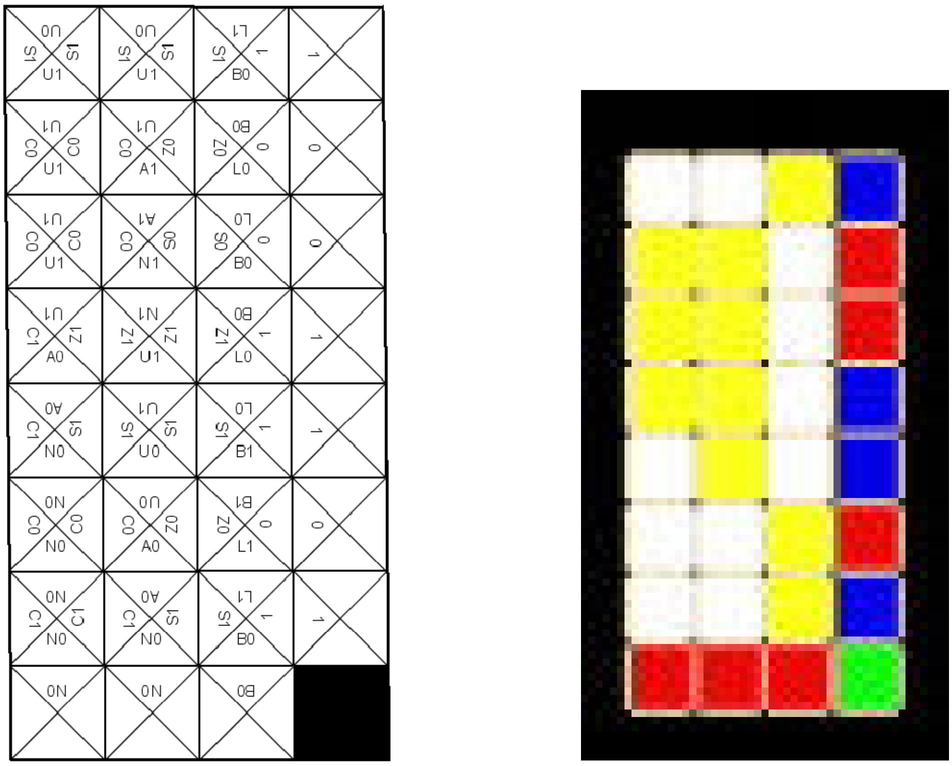

Figure 2 – Left: An example of error detection in the Hamming code 1011001 using the TAK tile module, the northern edge glue L1 shows an error in parity bit 0, the northern edge glues U0 show that parity bits 2 and 4 are correct.

Right: XGrow simulation of the example on the right. On the eastern edge of the module is the Hamming code with blue representing 1 and red representing 0. On the northern edge is the output showing erroneous parity bit 0 by yellow tile and correct parity bits 2 and 4 by white tiles.

Acknowledgements

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References

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[2] Goel, A. et al., *Error Free Self-Assembly using Error Prone Tiles.* Stanford University, Stanford, California.

[3] E. Winfree et al., *The XGrow Simulator*. http://dna.caltech.edu/Xgrow/xgrow_www.html.

[4] E. Winfree and R. Bekbolatov. *Proofreading tile sets: Error correction for algorithmic self-assembly*. In Proceedings of the Ninth International Meeting on DNA Based Computers. Madison, Wisconsin, June 2003.